A More Accurate Notation for Multiphonics Using Sideband Ratios

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ABSTRACT

By understanding saxophone multiphonics in terms of modulation synthesis sidebands it becomes possible to use the heterodyne interval structure to more clearly describe the sound we hear. In this case study, a method is developed by which the spectral data of multiphonics are analyzed for their harmonic content and then integrated into a unifying harmonic field. Applying developments in Just Intonation notation, a new multiphonic reference catalog is proposed in order to provide composers and performers with more acoustic accuracy than those currently available.

I. INTRODUCTION

There have been many texts written discussing multiphonics techniques for woodwind instruments since Bruno Bartolozzi's *New Sounds for Woodwind* was published in 1967. Famously, Bartolozzi's instructions to reproduce the sounds in his book were mostly unusable for other players. As researchers and performers have gained better understanding of the structure of the acoustical principles involved, multiphonics have gradually become a more standard technique among conservatory trained musicians. Nevertheless, there remains much work to be done in relating the actual harmonic structure of multiphonics to the confines of musical notation.

Composers seek to understand the instruments they compose with in order to predict the sounds that will be produced in performance. Of the many multiphonic catalogs today, most notate using vague indications of a pitch being "slightly" sharp or flat.[1][2][3] These descriptions, while not wholly incorrect, are problematic. The human ear can perceive very minute pitch alteration, and the difference of 10 cents(¢) in one voice can change the color of a chord. The most recent texts such as the Bärenreiter, *The Techniques of Oboe Playing*, have made significant steps in the direction of clarity, using spectrum analysis to find the frequencies present.[4] However, the Bärenreiter uses an 1/8th step Even Temperament grid – which is closer to reality than most texts – but because it is still an artificial grid in which a naturally occurring acoustic phenomena is quantized, it will still not truly represent the intervallic content of the sound.

In the 1980s, spectral information began to be utilized for extended techniques of orchestration. This opened new ways to manipulate sound information, and provided methods for musicians to work in terms of the structure of sound. To notate these sounds requires the use of tones smaller than the 12 pitches, which the majority of the last 200 years of musical theory has taken to be the whole. However, rational intervals do not fit comfortably into the Even Temperament system. Through the "tempering" of acoustically created intervals, pitches are forced to refer to an irrational grid. Still, this is the accepted system, and so we must work with the existing framework.

In this pilot study, we apply the recent advancement of intervallic notation, to the notation of saxophone multiphonics. New notation developed by Marc Sabat and Wolfgang von Schweinitz, based on the rational musical theories of Pythagoras and Ptolemy, through the techniques of Just Intonation, is introduced to more accurately describe the harmonic structure of multiphonics.[5][6][7]

A BRIEF INTRODUCTION TO THE PRINCIPLES OF JUST INTONATION

As opposed to Even Temperament (ET), the basis of Just Intonation (JI) is on the acoustic structure of sound. It is not restricted to a set number of pitches, but instead is notated in terms of ratios between frequencies.

In the natural harmonic series, each prime numbered harmonic and all multiples of that number, are equal to the same intervallic distance from the fundamental.

For example, with a fundamental frequency of 100 Hz, multiples by powers of 2 (200, 400, 800, 1600…etc.) will produce the overtone of the octave. Multiples by powers of 3 (300, 900, 2700 etc.) will produce the overtone of the fifth from the fundamental frequency, and so on. Similarly, this rule applies to the concept of undertones, where a frequency is divided by a power of 2 produces an octave below, and divided by powers of 3 produces the fifth below, and so on. For each prime number there is a unique interval distance.

Since the spectra of any sound may be harmonically described through the component sinusoidal waveforms, and acoustically created sounds are governed by the principles of the natural harmonic series, we can then assume that all acoustic intervals can be described in terms of natural harmonic intervals. We can depict the characteristic interval by factoring. Its combined prime number intervals describe the accumulated interval distance.

As an example, an interval of a 5:4 is a JI major third. From the frequency 100 Hz, 5:4 is equal to $100\left(\frac{5}{4}\right) = 100\left(\frac{5}{2^2}\right) = 125$ Hz.

The primes are 5 and 2. The numerator produces the fifth overtone at 500 Hz, and then the denominator lowers the pitch by 2^2, which is equal to two octaves.

As another example, two frequencies 100 and 187.5 Hz can be reduced to the ratio 15:8 or $100\left(\frac{15}{8}\right) = 100\left(\frac{3 \cdot 5}{2^3}\right) = 187.5$ Hz.

To calculate the distance of a ratio in cents, and apply this to our examples above we use the equation: $1200 \cdot \log_2\left(\frac{F_2}{F_1}\right)$.

What we find is that 5:4 = 386.3137¢, or in terms of ET four half-steps -13.6863¢. The interval of 3:2 = 701.9550¢, or seven ET half-steps +1.955¢, and 15:8 = 1088.27¢ or 11 half-steps -11.7313¢. Since 15:8 is equal to $\frac{3 \cdot 5}{2^3}$, we can also see that 15:8 will be equal to 11 half-steps plus 1.955¢ and minus 13.6863¢, or -11.7313¢ (we ignore powers of 2 since they do not have an offset in cents).

Pythagoras' theory of music was based on the ratio of the fifth 3:2. As a basis for pitches, he constructed a lattice of consecutive fifths, which reached new pitches at each step; this is a "spiral of fifths" rather than a "circle". In terms of our ET system, each consecutive fifth is then +1.955¢ sharper than the previous. So, for base pitch A4 = 440, a note B5, two consecutive fifths above is equal to $440\left(\frac{3^2}{2^2}\right) = 990$, A4 +1403.91¢, or B5 +3.91¢.

This system is called a 3-limit system since it is limited to the prime number series up to the prime number 3. Since each prime number overtone is a unique interval, it is not possible to describe intervals accurately of prime numbers higher than the limit of the system. Throughout history higher and higher prime numbers have been included into the harmonic field used by musicians. In general, each addition of primes is in terms of the Pythagorean basis of fifths.[7]

For example, Ptolemy's addition of the prime number 5 introduced the interval of the major and minor third to the system, and is defined as separated from the Pythagorean third by a *syntonic comma*, $\frac{81}{80}$, or ≈ 21.5¢. In terms of our previous notation, we could say that for frequency F: $F\left(\frac{5}{4}\right) = F\left(\frac{3}{2}^4 \cdot \frac{80}{81}\right)$.

In this study we apply a 23-limit system notation system to the analyses of multiphonics. Which, as we will show, provides the maximum possible accuracy with the minimum amount of deviation from the ET notation system.

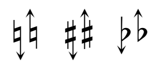
To notate the intervals, the *Extended Helmhotz-Ellis JI Pitch Notation* system developed by Marc Sabat and Wolfgang von Schweinitz will be used. A brief summary of the notation follows:

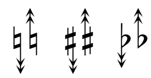
Extended Helmhotz-Ellis JI Pitch Notation by Marc Sabat and Wolfgang von Schweinitz


= equal temperament (EI) pitches


= Pythagorean intervals ≈ +/- 2¢ per step by fifth

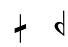
For Prime number intervals 5-23 all pitches are offsets from the Pythagorean interval

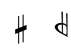

= raised/lowered by 1 syntonic comma $\frac{81}{80}$ ≈+/- 21.5¢

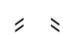

= raised/lowered by 2 syntonic commas


= raised/lowered by 1 septimal comma $\frac{64}{63}$ ≈ +/- 27.3¢


= raised/lowered by 2 septimal commas


= raised/lowered by 11-limit unidecimal quarter-tone $\frac{33}{32}$ ≈ +/- 53.5¢


= raised/lowered by 13-limit tridecimal third-tone $\frac{27}{26}$ ≈ +/- 65.3¢


= raised/lowered by 17-limit schisma $\frac{256}{255}$ ≈ +/- 6.8¢

= raised/lowered by 19-limit schisma $\frac{513}{512}$ ≈ +/- 3.4¢


= raised/lowered by 23-limit comma $\frac{736}{729}$ ≈ +/- 16.5¢

II. MATERIALS AND METHODS

Three multiphonics were chosen by my colleague Martin Posegga, who is a member of the Sonic Arts saxophone quartet in Berlin. The examples were selected as being representative of three main types of multiphonics. Although not a full descriptor of all possible colors of multiphonics, they represent three combinations of normal and altered embouchure and fingering performance techniques.

The first multiphonic (M1), is an example of a wide spectrum chord. The performance of M1 requires no alteration of embouchure technique. It is an example of the basic, easy level multiphonic that is produced swiftly with the correct fingering and normal embouchure and air pressure. The timbre of the chord is strong in higher partials, which gives a more dissonant, oboe-like sound.

Multiphonic #2 (M2) is an example of the "sub-tone" type multiphonic. When played softly, the result is close interval diads. The performance requires a "bending" of the embouchure.[3] The sub-tone timbre is frequently used by jazz saxophonists, and is characterized by a slightly flatted pitch and smooth airy tone. When played with more force, higher partials enter the chord above the lower diad.

Multiphonic #3 (M3) is an example of a technique, which could be thought of as being the opposite of M1. The fingering is of a typical tempered pitch, which when played with an alternate embouchure produces a multiphonic. The embouchure technique is similar to that of a beginner saxophonist, but with control, a wide octave diad appears. The dynamic range of this multiphonic is very limited, and can be played only at a very low volume.

These three multiphonics were then recorded several times on three different tenor saxophones, using the same mouthpiece and reed. The instruments used were: the Yamaha *Custom EX*, Selmer *Super Action II*, and Buffet-Crampon *Prestige S1*. The mouthpiece was a Vandoren *T20*, with a Vandoren *Classic* 3.5 reed.

The recording device used was the digital Zoom H2 Handy Recorder. The samples were recorded in an apartment with a normal amount of reverberation and no additional sound baffling. The microphone was stereo, and set to be in directional mode.

Each multiphonic was recorded with each instrument once tuned to A440, and again tuned to A443. The recordings were then transferred to a computer and sliced into smaller samples to be analyzed.

The analysis was done in several steps using self-designed Matlab scripts. The signal was first analyzed using Matlab's FFT function, and then the results were filtered by amplitude cut-off frequency calculated from the noise elements of the recording. Peak frequencies were then located by scanning the data for the highest point within a look range of 70-90 cents, depending on the amount of bleeding in the FFT. Depending on the length of recording, the frequency resolution of the analysis varied from 1 Hz to 0.1 Hz.

First experiments in analyzing the tonal structures of the multiphonics applied an exhaustive iteration script to methodically look for the lowest possible ratios between all found peaks within a tolerance range. Later, once it was determined that the structures of the multiphonics were almost entirely heterodyne lattices, only the first two pitches were examined for the best possible ratio. The script uses the GCD Matlab function to find each possible ratio at a resolution of 0.01 Hz.

The list of possible ratios was compiled, and then each ratio was compared and sorted by "Harmonic Distance" (HD) as proposed by James Tenney, where for ratio A:B:

$$HD = \log_2(a \cdot b)$$ [8]

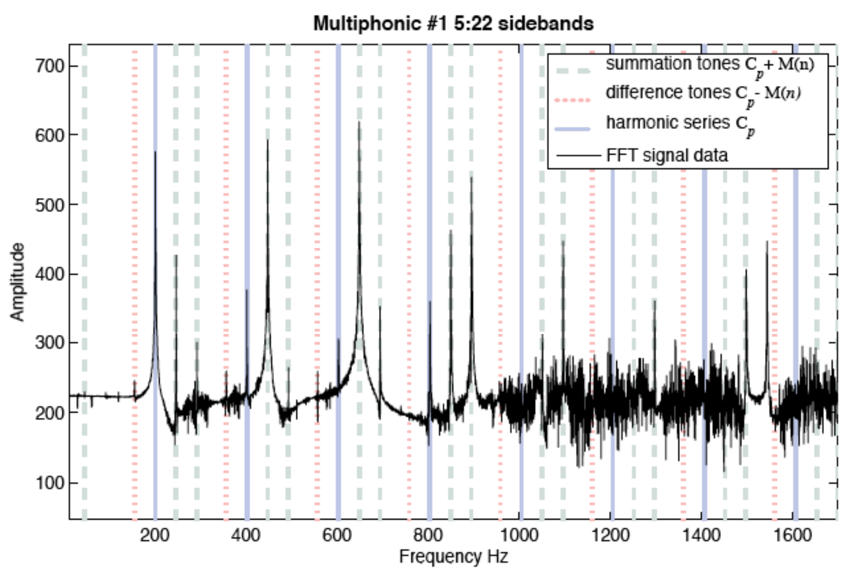


Figure 3.1: Frequency Spectrum of Multiphonic #1

III. RESULTS AND APPLICATION OF ANALYSIS

In his book, *Fundamentals of Musical Acoustics*, Arthur Benade published one of the clearest explanations of multiphonics. He shows that the pitches in the examples he studied are consistently within the lattice of heterodyne intervals (i.e. summation and difference tones).[9] While the study here reinforces Benade's research, it was found that the principle of frequency modulation may be an additional support in unraveling the harmonic structure of multiphonics into a generalized theory which could be applied more predictively.

In the case of the first multiphonic sample (M1) played on the Yamaha saxophone tuned at A440, it was determined by the ratio finding algorithm that the closest lowest HD ratio between pitches F1 and F2 (200.91 and 246.63 Hz) to be 27:22 within a tolerance range of 2¢. The algorithm finds this by looking above and below each frequency at a maximum deviation of 2¢, in this case the deviation was less than 0.1¢.

Comparing the frequencies in Hertz through ratios we can confirm that since F1 is 200.91 Hz, and the ratio relationship between F2:F1 is 27:22, that $F_2 = F_1\left(\frac{27}{22}\right)$= 246.63 Hz. The distance between F1 and F2 is 5:22, or $F_2 = F_1 + F_1\left(\frac{5}{22}\right)$.

Considering the heterodyne relationship in terms of sidebands in amplitude modulation synthesis(AM), the sideband equation predicts where the difference and summation tones will appear in the signal:

$$SB_{np} = C_p \pm M(n)$$

where sideband pairs SB at index number *n* and carrier partial *p*, is equal to the modulator times the index number added and subtracted to the carrier at partial *p*. [10]

If we consider F1 to be the carrier frequency, and C = 200.9 Hz, it follows that since $F_1\left(\frac{5}{22}\right)$ is equal to the distance F2-F1, that for modulator M:

$$M = F_1\left(\frac{5}{22}\right) = 45.73 \text{ Hz.}$$

Yamaha: Multiphonic #1
A4 hz = 440
peak find range = 75¢
ratio tolerance = 2¢
amplitude cutoff = 275
sample length = 154313
bin resolution = 0.29 Hz

	Hz.	Pitch	Amp.
1	200.91	g3 42.81¢	575.8
2	246.63	b3 -2.18¢	426.7
3	292.35	d4 -7.73¢	299.5
4	401.81	g4 42.81¢	377.5
5	447.53	a4 29.40¢	592.7
6	602.71	d5 44.77¢	304.5
7	648.44	e5 -28.63¢	619.3
8	694.16	f5 -10.7¢	354.1
9	803.62	g5 42.81¢	361.4
10	849.34	ab5 38.62¢	463
11	895.07	a5 29.40¢	538.8
12	1050.25	c6 6.19¢	311.7
13	1095.9	db6 -20.03¢	446.8

Figure 3.2 Peak Frequencies found in M1

Once we have calculated the Carrier-Modulator ratio (C:M), we can apply the ratio, numerator (C) and denominator (M), in place of the frequencies, and using this ratio calculate the SB sum and difference tones. In this way we begin to look at the harmonic interval content rather than the absolute frequency. By abstracting the frequencies into ratios we are able to describe the relative structure of the multiphonic in its own terms.

22(1) - 5(1) = 17
22(1) + 5(1) = 27
22(1) + 5(2) = 32
22(2) - 5(1) = 39
22(2) + 5(1) = 49
22(2) + 5(2) = 54
22(3) - 5(1) = 61
22(3) + 5(1) = 71
22(3) + 5(2) = 76
22(4) - 5(1) = 83
22(4) + 5(1) = 93
22(4) + 5(2) = 98
22(5) - 5(1) = 105
22(5) + 5(1) = 115
22(5) + 5(2) = 120
22(6) - 5(1) = 127
22(6) + 5(1) = 137
22(6) + 5(2) = 142
22(7) - 5(1) = 149
22(7) + 5(1) = 159
22(7) + 5(2) = 164
22(8) - 5(1) = 171
22(8) + 5(1) = 181
22(8) + 5(2) = 186
22(9) - 5(1) = 193
22(9) + 5(1) = 203
22(9) + 5(2) = 208
22(10) - 5(1) = 215

Figure 3.3 Calculations for sidebands ratios for up to the tenth partial of F1

The SB calculation in figure 3.3, shows the first two upper sidebands above the carrier series at a C:M ratio of 22:5. It was found that the difference tones are much lower than the sum tones in amplitude. This may be due to the fact that the relative strength of M is quite low. Applying the calculation to the signal data, it has proven sufficient to look two sideband indices above C at partial *p* and only one index below.

SB calculation	Fundamental Series			Pitches Found in FFT Analysis			
		Hz	midi	Hz	midi	pitch	amp.
	55	502.26	71.29				
22(2) + 5(2) = 54	**54**	493.13	70.97	493.26	70.98	B4 -2.18¢	263.91
	53	484.00	70.65				
	52	474.87	70.32				
	51	465.74	69.98				
	50	456.60	69.64				
22(2) + 5(1) = 49	**49**	447.47	69.29	447.54	69.29	A4 29.40¢	592.69
	48	438.34	68.93				
	47	429.21	68.57				
	46	420.07	68.20				
	45	410.94	67.82				
22(2) = 44	**44**	401.81	67.43	401.81	67.43	G4 42.81¢	377.52
	43	392.68	67.03				
	42	383.55	66.62				
	41	374.41	66.21				
	40	365.28	65.78				
22(2) - 5(1) = 39	**39**	356.15	65.34	356.37	65.35	F4 35.05¢	257.13
	38	347.02	64.89				
		…					
	33	301.36	62.45				
22(1) + 5(2) = 32	**32**	292.23	61.91	292.36	61.92	D4 -7.73¢	299.51
	31	283.09	61.37				
	30	273.96	60.80				
	29	264.83	60.21				
	28	255.70	59.60				
22(1) + 5(1) = 27	**27**	246.57	58.97	246.63	58.98	B3 -2.18¢	426.74
	26	237.43	58.32				
	25	228.30	57.64				
	24	219.17	56.93				
	23	210.04	56.20				
C	**22**	200.91	55.43	200.91	55.43	G3 42.81¢	575.87
	21	191.77	54.62				
	20	182.64	53.78				
	19	173.51	52.89				
	18	164.38	51.95				
22(1) - 5(1) = 17	**17**	155.25	50.96	155.46	50.99	D#3 -1.09¢	228.90
	16	146.11	49.91				
	15	136.98	48.80				
		…					
	6	54.79	32.93				
M	**5**	45.66	29.78	45.73	29.80	F#1 -19.73¢	229.85
	4	36.53	25.91				
	3	27.40	20.93				
	2	18.26	13.91				
Fundamental	**1**	9.13	1.91				

Figure 3.4: Multiphonic #1 in terms of fundamental series $F_1\left(\frac{1}{22}\right) = F_2\left(\frac{1}{27}\right)$

In finding a ratio between F1 and F2, we have essentially unified the frequencies by fitting them into a harmonic series grid. This exposes the sidebands' natural tonal order. In the case of M1, using a F1:F2 ratio of 27:22, the ratio tells us that the fundamental *Grundton* (G) is equal to:

$$G = F_1\left(\frac{1}{22}\right) = F_2\left(\frac{1}{27}\right) \text{ or } 9.1344 \text{ Hz.}$$

Using fundamental pitch G as represented by ratio 1:1, we can then fit the sideband ratio calculations, and rationally describe the multiphonic harmony.

Figure 3.4 illustrates the first nine pitches found in M1 as defined by sideband ratios within the fundamental harmonic series. The graph in figure 3.1 shows the FFT signal with the sideband ratios marked in vertical lines, as defined by the chart in figure 3.4.

What was discovered when comparing the results for M1, is that the ratios are fairly consistent between tunings on each instrument. The Yamaha and Selmer models appear to have linear relationships between ratios. However, there exists a distance of ca. 50¢ between their respective performances of M1. The Buffet-Crampon saxophone clusters near the Yamaha results, but there is more variation in pitch. (Figure 3.5)

Naturally, the accuracy of the F2:F1 ratio will affect the resulting rational heterodyne lattice. This accuracy will be reflected in the deviation of pitch calculations applied to other FFT data. However, Figure 3.5 suggests that the ratio 43:35 is the average for Yamaha and Buffet-Crampon samples, with 27:22 a close neighbor (and simpler prime ratio).

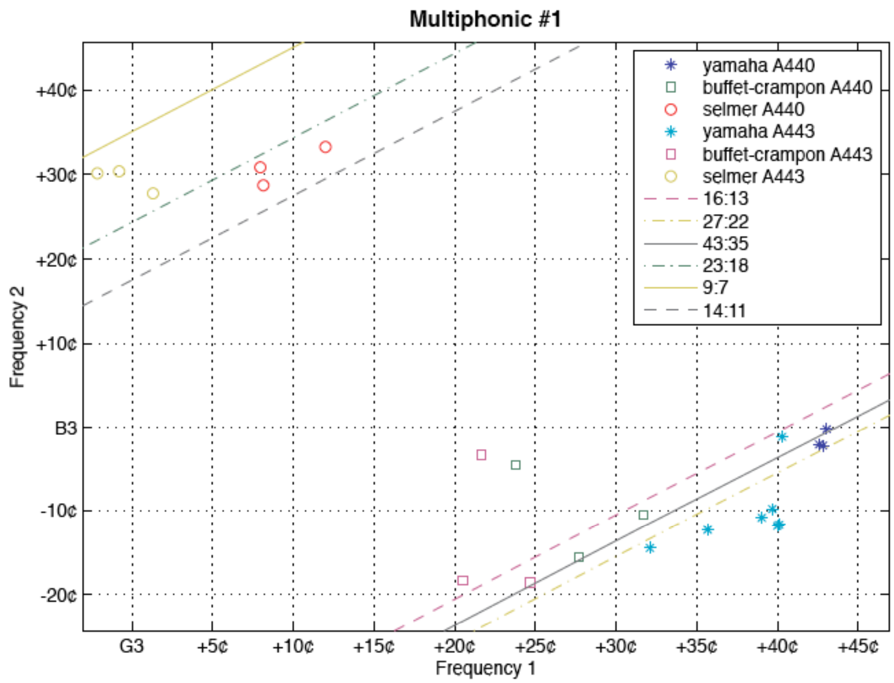


Figure 3.5: Comparison of lowest term ratios found between F2:F1 with Yamaha, Selmer and Buffet-Crampon Tenor Saxophones, at tunings A440 and A443

As mentioned above, what is immediately apparent looking at the chart of F1:F2 ratios in figure 3.5, is the separation between the Selmer and Yamaha and Buffet-Crampon saxophones. The Selmer is quite consistent along the 23:18 axis, and the 27:22 axis is the closest average for all of the Yamaha results. The Buffet-Crampon results lay close to the 27:22 also, off by ca. 2-4¢, and closest to the 43:35 axis. The Yamaha recordings at A440 are also close to the 43:35, which could also be used as a base ratio with little accuracy issues, as well as 16:13, which is close to an average ratio for the Buffet-Crampon, and just 1-2¢ off for the upper F2 range of the Yamaha.

It is notable, that the figure 3.5 also shows that variation in pitch for F1 is greater than that of F2.

Figures 3.6 - 3.7 show the application of the 27:22 F1:F2 ratio to the FFT data from M1 on each instrument. It is visible that the Selmer has a slightly different shape to its spectrum, and a significant suppression of the F1 overtone series. Even so, it closely follows the sideband pattern found in the Yamaha and Buffet-Crampon models. In normal wind-instrument performance it is typical to need alternate fingers to play in tune. The graph in 3.7 illustrates the fact that the structure of the multiphonic is in fact quite close in structure.

Comparing the interval ratios in terms of cents: 27:22 = 354.6¢; 43:35 = 356.4¢; 23:18 = 424.4¢. The difference between 23:18 and 27:22 = 69.8¢.

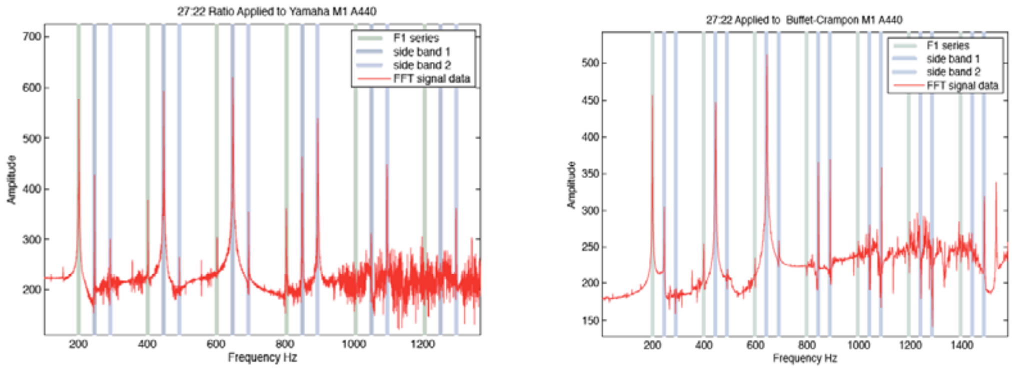


Figure 3.6: M1 F1:F2 ratio 27:22 (C:M ratio 22:5) Yamaha and Buffet-Crampon Saxophones

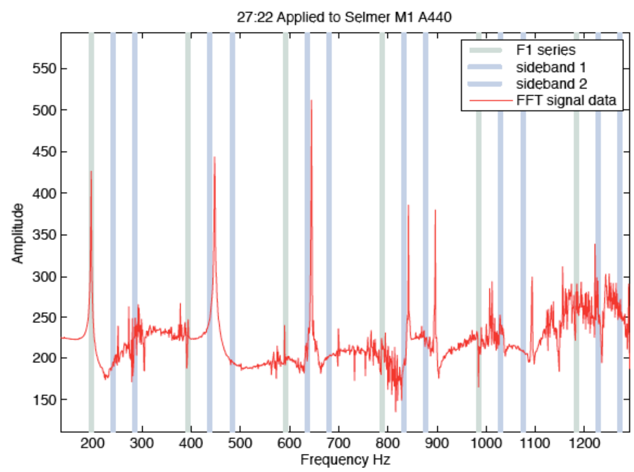


Figure 3.7: M1 ratio 27:22 Selmer Saxophone – Note: the ratio is slightly too small, which offsets the results; however the structure of the multiphonic is very similar.

Due to the relative nature of harmonics, it is useful in analysis to approach a given harmony on its own terms. In the case of an acoustically created harmony such as this, the characteristic ratios are the relations between the component frequencies. Analysis from a relative perspective simplifies the interval. However, working within an ensemble musical situation it becomes necessary to relate pitches to the grid of tempered pitches. Placing the interval structure in relation to the ET grid, an agreed upon pitch series can be used to move between harmonies.

Figure 3.8 shows the results from the ratio finding algorithm for ratios between F1:A4 (440 and 443 respectively) and F2:A4. The ET pitches G3 and B3 are shown as solid lines. It is interesting to observe that the deviation between the Selmer F1 443 pitches cluster around ET pitch G3, while the Yamaha F2 440 pitches cluster near ET B3.

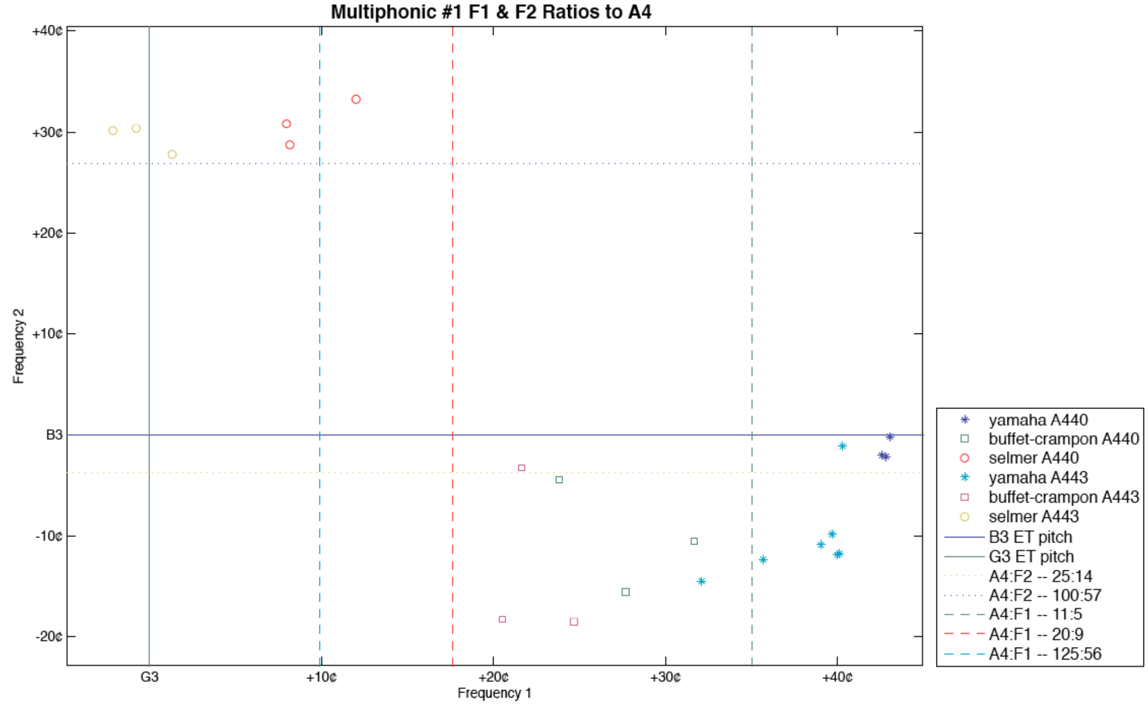


Figure 3.8 Ratios from F1 and F2 to A4
F1 ratios are shown vertically and F2 ratios horizontally

In terms of A4, the closest JI interval to B3 is the Pythagorean 9:8, in terms of ET the pitch would be B3 +4¢. However, looking at the data in Figure 3.8, we see that the majority of the F2 pitches are located just flat of the ET B3 line. Therefore, positioning the chord at 9:8 would offset all of the intervals by ca. 6¢. This is still much more accurate than using 1/8th (25¢) step ET notation, as found in the Bärenreiter Oboe text. One might argue that because the saxophone is constructed to play ET music, it is reasonable to use ET pitches other than A4 as a reference in this case.

Figure 3.9 shows the multiphonic as notated in the *Extended Helmholtz-Ellis* system. Using an ET pitch as our reference greatly reduces necessary accidentals. Figure 3.10 shows an alternate possibility for notating M1. The accidentals are the product of the primes in the chord.

For figure 3.9 where F2 = ET B3, we can express pitches 1-7 as follows:

$$F_1 = F_2\left(\frac{22}{27}\right) = B3\left(\frac{2\cdot 11}{3^3}\right)$$

$$F_2 = B3$$

$$F_3 = F_2\left(\frac{32}{27}\right) = B3\left(\frac{2^5}{3^3}\right)$$

$$F_4 = F_2\left(\frac{49}{27}\right) = B3\left(\frac{7^2}{3^3}\right)$$

$$F_7 = F_2\left(\frac{71}{27}\right) = B3\left(\frac{71}{3^3}\right)$$

$$F_7 - 24.6¢ = F_2\left(\frac{70}{27}\right) = B3\left(\frac{2\cdot 5\cdot 7}{3^3}\right)$$

$$F_7 + 24.2¢ = F_2\left(\frac{72}{27}\right) = B3\left(\frac{2^3\cdot 3^2}{3^3}\right)$$

The interval 71:27 is outside of our prime limit system so it has been replaced by the closest possible accidental within the series, 72:27. It is in these cases where the system breaks down. However, since the error margin is less than 25¢ only in the one note, it is still much more accurate than 1/8th tone ET notation. Diamond note-heads represent the most prominent peak pitches.

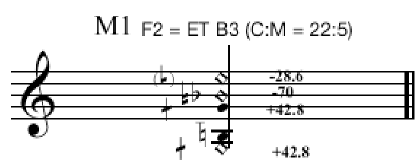


Figure 3.9: The notation of M1 using ET B3 as F2

In the figure 3.10, we see an example of why it is practical to use one close interval to A4 rather than another. The number of primes in a given pitch will determine how many accidentals will need to be used, and how difficult the notation will be to read. In this example, the A4:F2 ratio 25:14 (while being on average 2-3¢ more accurate in placement compared to F2=ET B3) introduces the prime numbers 5 and 7 which where not present in the notation in the F2=ET B3 version. So for 25:14:

$$F_1 = A4\left(\tfrac{14}{25}\right)\cdot\left(\tfrac{22}{27}\right) = A4\left(\frac{2^2\cdot 7\cdot 11}{3^3\cdot 5^2}\right)$$

$$F_2 = A4\left(\tfrac{14}{25}\right) = A4\left(\frac{2\cdot 7}{5^2}\right)$$

$$F_3 = A4\left(\tfrac{14}{25}\right)\cdot\left(\tfrac{32}{27}\right) = A4\left(\frac{2^6\cdot 7}{3^3\cdot 5^2}\right)$$

$$F_4 = A4\left(\tfrac{14}{25}\right)\cdot\left(\tfrac{49}{27}\right) = A4\left(\frac{2\cdot 7^3}{3^3\cdot 5^2}\right)$$

$$F_7 = A4\left(\tfrac{14}{25}\right)\cdot\left(\tfrac{71}{27}\right) = A4\left(\frac{2\cdot 7\cdot 71}{3^3\cdot 5^2}\right)$$

$$F_7 - 24.6¢ = A4\left(\tfrac{14}{25}\right)\cdot\left(\tfrac{70}{27}\right) = A4\left(\frac{2^2\cdot 5\cdot 7^2}{3^3\cdot 5^2}\right)$$

$$F_7 + 24.2¢ = A4\left(\tfrac{14}{25}\right)\cdot\left(\tfrac{72}{27}\right) = A4\left(\frac{2^3\cdot 3^2}{3^3\cdot 5^2}\right)$$

Figure 3.10: The notation of M1 using A4:F2 = 25:1

Multiphonic #2 (M2) in support of our analysis from M1 follows the same sideband pattern, this time with a different F1:F2 interval. The spectrum differs also from M1 in that the noise element is stronger, the pitches are more diffused, and the difference tones are no longer present to the degree found in M1.

In terms of pitch placement, data shows again that the Selmer saxophone is offset from the Yamaha and Buffet-Crampon saxophones by ca. 50¢. In M2 the Selmer is sharp in comparison, where in M1 it was flat.

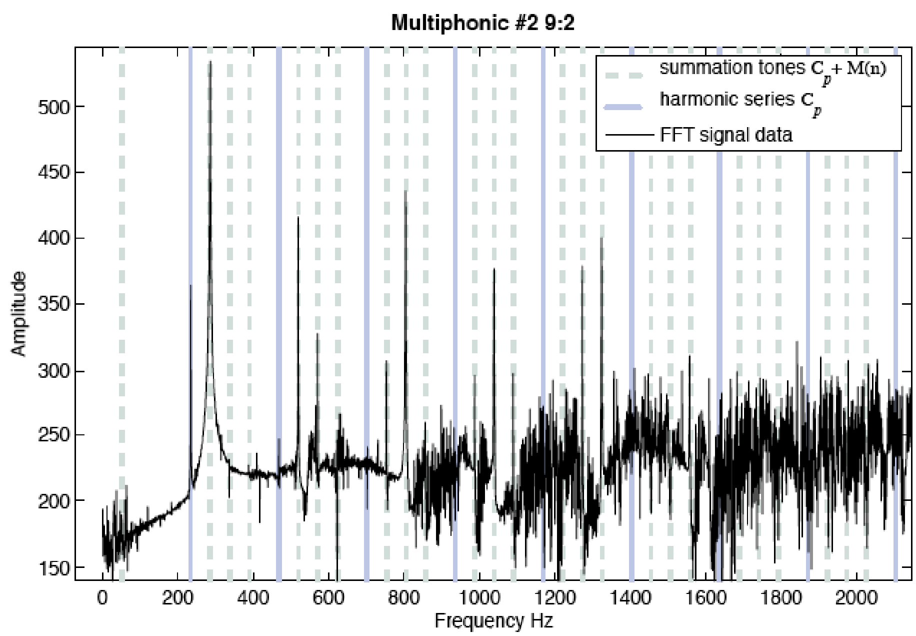


Figure 3.11: Multiphonic #2 with sidebands F2:F1 = 11:9, C:M = 9:2

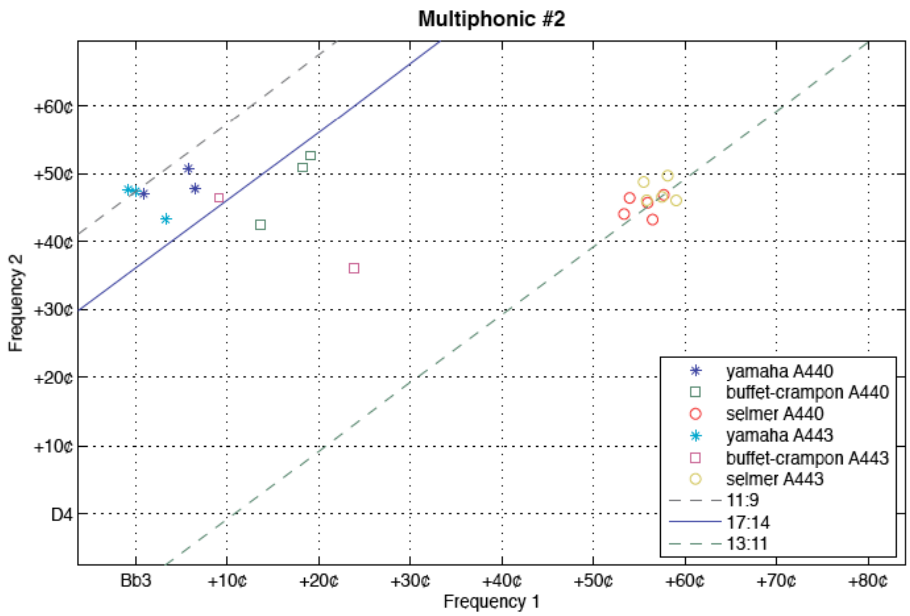


Figure 3.12: Multiphonic #2 F2:F1 ratios

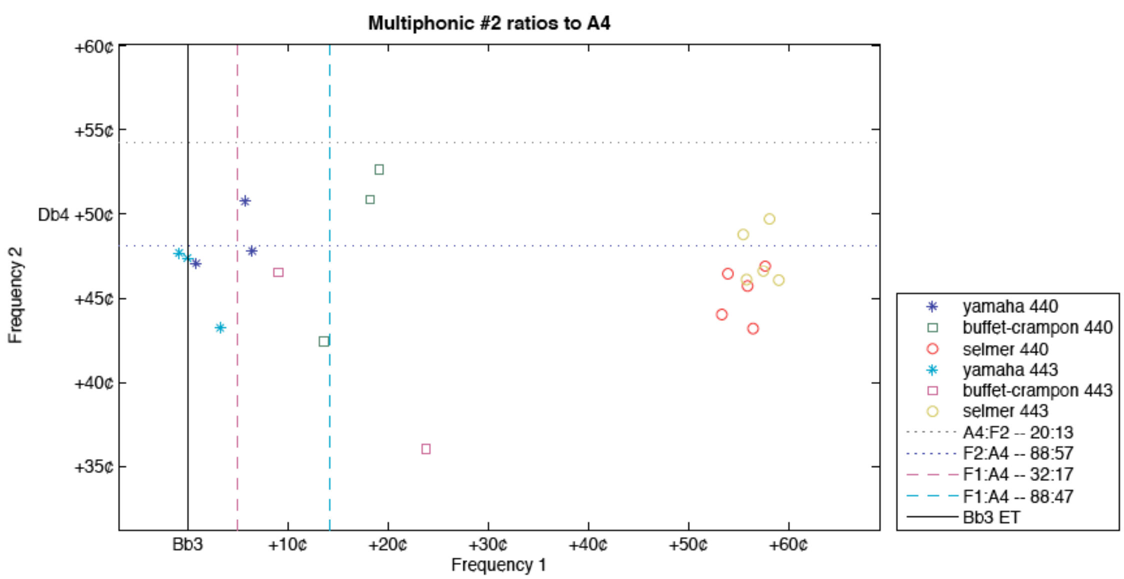


Figure 3.13 Multiphonic #2 ratios to A4

The placement of F1 and F2 in terms of A4 for M2 is illustrated in figure 3.13. Again, data shows that the pitches tend to cluster around ET pitches. In this case F1 is close to Bb3. The best average for the Yamaha saxophones is A4:F1 ratio 32:17, which is notable because the interval 17:16 is equal to 105¢, only 5¢ above the ET half-step.

Applying the A4:F1 ratio 32:17, we transpose the structure of the multiphonic with C:M ratio 9:2 through the following series:

$$F_1 = A4\left(\frac{17}{32}\right) = A4\left(\frac{17}{2^5}\right)$$

$$F_2 = A4\left(\frac{17}{32}\right)\cdot\left(\frac{11}{9}\right) = A4\left(\frac{11\cdot 17}{2^5\cdot 3^2}\right)$$

$$F_3 = A4\left(\frac{17}{32}\right)\cdot\left(\frac{20}{9}\right) = A4\left(\frac{5\cdot 17}{2^3\cdot 3^2}\right)$$

$$F_4 = A4\left(\frac{17}{32}\right)\cdot\left(\frac{22}{9}\right) = A4\left(\frac{11\cdot 17}{2^4\cdot 3^2}\right)$$

$$F_5 = A4\left(\frac{17}{32}\right)\cdot\left(\frac{29}{9}\right) = A4\left(\frac{17\cdot 29}{2^5\cdot 3^2}\right)$$

$$F_6 = A4\left(\frac{17}{32}\right)\cdot\left(\frac{31}{9}\right) = A4\left(\frac{17\cdot 31}{2^5\cdot 3^2}\right)$$

$$F_7 = A4\left(\frac{17}{32}\right)\cdot\left(\frac{40}{9}\right) = A4\left(\frac{5\cdot 17}{2^2\cdot 3^2}\right)$$

$$F_8 = A4\left(\frac{17}{32}\right)\cdot\left(\frac{51}{9}\right) = A4\left(\frac{17^2}{2^5\cdot 3}\right)$$

This transposition works well as an average between the Yamaha and Buffet-Crampon saxophones. Because the intervals of prime 17 and 19, are very close to the Pythagorean (ca. ±3.4¢), they are notated here with 17 and 19 intervals filtered.

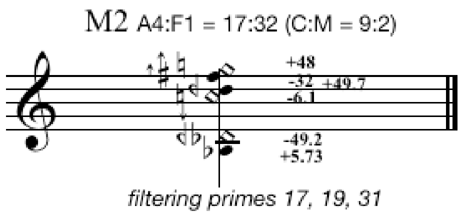


Figure 3.14: Multiphonic #2 notation

Alternatively, the F2:A4 interval 88:57 is the average ratio for all M2 results. Using a F2:F1 ratio of 17:14, which is another close interval for Yamaha and Buffet-Crampon instruments, we can transpose M2's interval structure through the following modulation:

$$F_1 = A4\left(\frac{13}{20}\right)\cdot\left(\frac{14}{17}\right) = A4\left(\frac{2\cdot 7\cdot 13}{2^2\cdot 5\cdot 17}\right)$$

$$F_2 = A4\left(\frac{13}{20}\right) = A4\left(\frac{13}{2^2\cdot 5}\right)$$

$$F_3 = A4\left(\frac{13}{20}\right)\cdot\left(\frac{31}{17}\right) = A4\left(\frac{13\cdot 31}{2^2\cdot 5\cdot 17}\right)$$

$$F_4 = A4\left(\frac{13}{20}\right)\cdot\left(\frac{34}{17}\right) = A4\left(\frac{2\cdot 13}{2^2\cdot 5}\right)$$

$$F_3 = A4\left(\frac{13}{20}\right)\cdot\left(\frac{36}{17}\right) = A4\left(\frac{3^2\cdot 13}{5}\right)$$

$$F_3 = A4\left(\frac{13}{20}\right)\cdot\left(\frac{31}{17}\right) = A4\left(\frac{2^2\cdot 3\cdot 13}{5\cdot 17}\right)$$

The notation results are shown in figure 3.15. On the left is an example including the primes 17 and 19. On the right 17 and 19 have been filtered for clarity. The cents notation is unaltered, and so is off from the notion by less than 7¢.

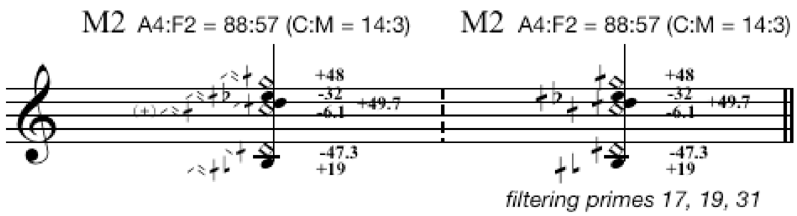


Figure 3.15: Multiphonic #2 notation

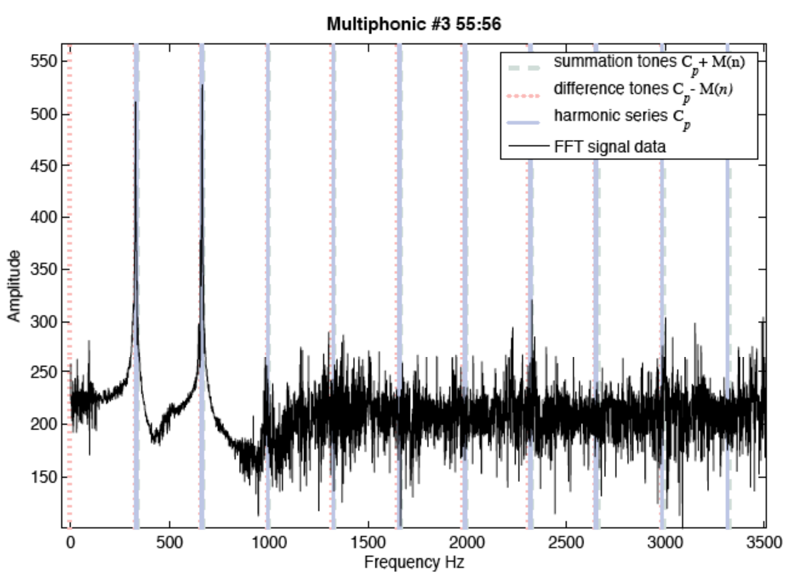


Figure 3.16 Multiphonic #3 FFT signal, C:M ratio 55:56 (F2:F1 ratio 112:55)

The results from multiphonic #3 (M3) are quite different from the others. The nature of the sound is almost completely noise except for the two main pitches, which form a wide major seventh. The analysis was more difficult for M3 as the pitches are unstable, and tend to waver in and out. Samples of different lengths of up to five seconds were used for the FFT.

Because the interval is close to the octave, the ratios produced through the sideband calculation are near to a normal harmonic series. As is visible in figure 3.16 the difference and summation tones directly link to the fundamental series of F1. This close proximity to the octave also puts the frequencies within critical band distance of upper partials effectively destabilizing the energy at these frequencies.

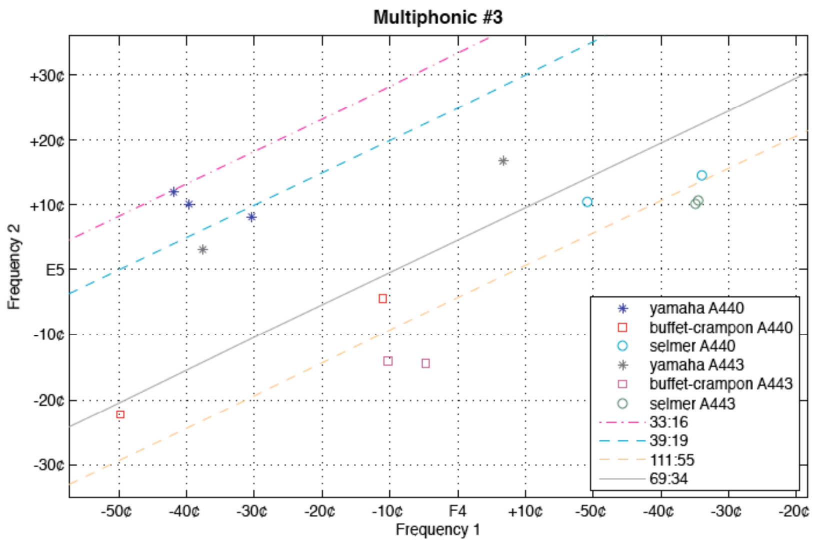


Figure 13.17: Multiphonic #3 ratios

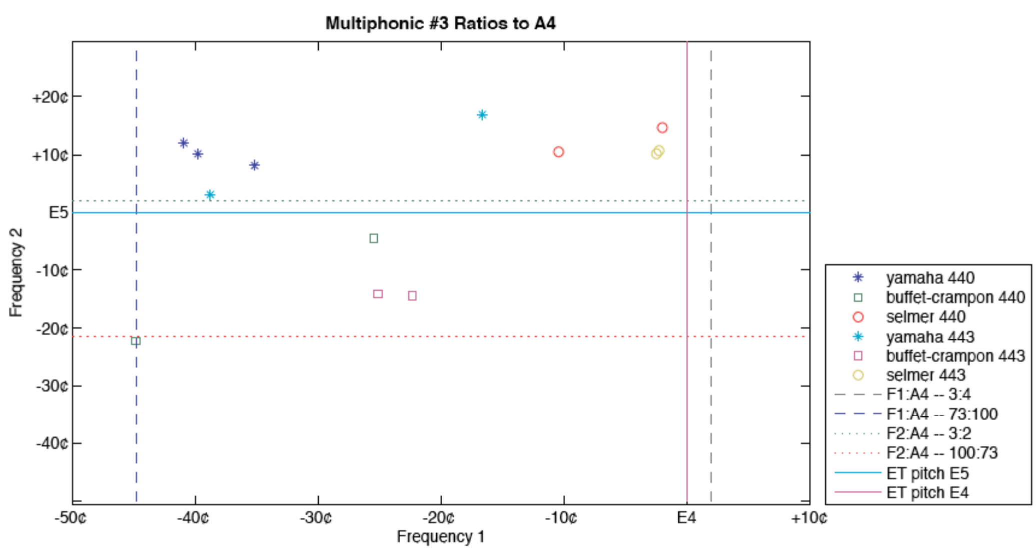


Figure 13.18: Multiphonic #3 ratios to A4

The average ratio F2:A4 in multiphonic #3 is close to 3:2, and for notation of the Yamaha, a C:M ratio of 19:20 was used. The interval of prime 19 was included for accuracy.

$$F_1 = A4\left(\frac{3}{2}\right)\cdot\left(\frac{19}{39}\right) = \left(\frac{3\cdot 19}{2\cdot 3\cdot 13}\right)$$

$$F_2 = A4\left(\frac{3}{2}\right)$$

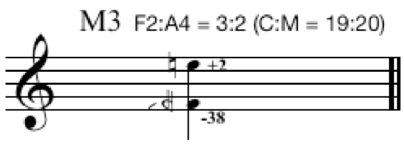


Figure 3.19: Multiphonic #3 notation

IV. CONCLUSIONS

As we have shown in this study, it is possible to accurately notate multiphonics within a closed ET system. It is a pilot study for future catalogs of multiphonics. Still there remain many possible multiphonics that have not been analyzed using this method, and while our study results remained consistently practical using this SB based method, it is possible that with more testing other structures could emerge.

The notation of timbre may be a useful addition to a new catalog of multiphonics. This could be accomplished by tracking the component overtones of each peak note, and measuring the strength of harmonics. Alternate note-heads could be used to indicate relative timbre of pitches.

From the results of the analysis, it appears to indicate that the Yamaha saxophone is potentially a more linear, and in tune instrument in comparison with the Buffet-Crampon and Selmer tenor saxophones. Due to normal intonation issues in woodwind instruments potential further research should examine the use of alternate fingerings to achieve the same multiphonic across different models of saxophones.

If the Yamaha instrument does prove to be more consistent and reliable between versions of the same make and model, it may be appropriate to construct a catalog specifically for the Yamaha instrument. The results from the Selmer saxophone seem to imply a consistent parallel to the results in the Yamaha instrument, and so if the above-mentioned fingering alteration is possible, it could mean that the results for both instruments could be brought together successfully. This would make the catalog for the Yamaha valid also for the Selmer instruments.

The application of sideband equation from modulation synthesis points to another possible area of research which might shed light on the physics involved in multiphonic production. From Benade's writings on wind instrument construction, it remains a possibility that through manipulation of the reed vibration an "air leak" is created. This changes the vibration pattern in a non-linear way such that upper partials may take hold without the presence of the fundamental pitch. Thus it seems plausible that the reed is vibrating at the rate of the modulation frequency.

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